MATH 103 Fall 2023: Group Assignment 4
Due Thursday, October 5, 2023 at 4:30 PM

Instructions

- Select your group members on Crowdmark. You can only add members who have not already been added to another group. Once a student is added to a group, the submission page is shared among the group members and any member can submit work on behalf of the group. Once a group assignment has been submitted, students will **not** be able to edit the group members. Any single member of the group can submit on behalf of the group and a single group assessment will be graded. **To avoid overwriting submissions, we recommend that only one group member submits.**
- Complete the following questions, writing up complete solutions, showing your work.
- There are four submission slots on Crowdmark: **Q1**, **Q2**, **Q3**, and **Q4**. Please upload your solutions into the appropriate slots.

Problems

1. Each of the following augmented matrices in row-echelon form corresponds to a linear system of equations in x, y and z (in that order). State how many solutions each system of equations has. **You do not need to solve the systems.**

   (a) $\left[\begin{array}{ccc|c} 3 & 5 & -9 & 7 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 4 & 0 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 3 & 2 & 6 & -5 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$

2. Each of the following augmented matrices in row-echelon form corresponds to a system of equations in x, y and z (in that order). For each matrix, state the solution to the corresponding system of equations, if it exists. If no solution exists, explain why. If there are an infinite number of solutions, write your answer in terms of a parameter.

   (a) $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 7 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 2 & 7 & -3 \\ 0 & 3 & 5 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right]$

3. FoodMart sells peanuts for \$11 per kilogram, cashews for \$33 per kilogram and almonds for \$16 per kilogram. They want to mix these nuts together so as to obtain 100 kg of mixed nuts worth \$18 per kilogram. They want the mass of the peanuts in the mix to be twice the mass of the cashews. What mass of each nut should be used in the mix?

   (a) Give a system of linear equations that would need to be solved to solve this problem. Clearly indicate what each variable represents.
   (b) Give the corresponding augmented matrix. **You do not need to solve the problem.**

4. Tickets to a play cost \$25 for adults, \$13 for children and \$19 for seniors. In total, 200 tickets were sold for a total revenue of \$4010. The number of children's tickets sold was 15 more than the number of senior's tickets sold. How many tickets of each type were sold?

   (a) Give a system of linear equations that would need to be solved to solve the problem. Clearly indicate what each variable represents.
   (b) Solve the problem using matrices and row reduction.

MATH 103 Fall 2023: Written Assignment 4
Due Monday, October 16, 2023 at 8:20 AM

Instructions

- There are five submission slots on Crowdmark: **Q1**, **Q2**, **Q3**, **Q4**, and **Q5**. Please upload your solutions into the appropriate slots.
- This assignment covers sections 3.3-3.5 (inclusive).

Problems

1. Each of the following augmented matrices in row-echelon form corresponds to a linear system of equations. If there are three variables, then they are x, y and z (in that order). If there are 4 variables, then they are w, x, y and z (in that order). State how many solutions each system of equations has. **You do not need to solve the systems.**

   (a) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$ (b) $\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 2 & 6 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

2. Each of the following augmented matrices in row-echelon form corresponds to a system of equations in x, y and z (in that order). For each matrix, state the solution to the corresponding system of equations, if it exists. If no solution exists, explain why. If there are an infinite number of solutions, write your answer in terms of a parameter.

   (a) $\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 3 & -9 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. Animal feed is to be made from corn, soybeans and cottonseed. Determine how many units of each ingredient are needed to make a feed that supplies 3300 units of fiber, 2300 units of protein, and 2950 units of fat, given that one unit of each ingredient provides the number of units shown in the table below. For example, one unit of corn provides 15 units of fiber, 30 units of protein and 20 units of fat.

	Units of Fiber	Units of Protein	Units of Fat
Corn	15	30	20
Soybeans	30	40	10
Cottonseed	45	10	40

   (a) Give a system of linear equations that would need to be solved to solve the problem. Clearly indicate what each variable represents.
   (b) Give the corresponding augmented matrix.
   (c) Solve the problem using row reduction.

4. For each of the following sets of conditions, determine all possible values of a, b, and c (if any) such that the quadratic function $f(x) = ax^2 + bx + c$ satisfies the given conditions. Use matrices and row reduction. If there are infinitely many solutions, state your solution using a parameter.

   (a) $f(1) = 2$ and $f(0) = 3$.

(b) $f(1) = f(-1) = 3$, $f(2) = 0$ and $f(0) = 5$.

5. Skye has 500 coins that are nickels, dimes and quarters. The total value of their coins is \$40.

   (a) Give a system of linear equations that would need to be solved to solve this problem. Clearly indicate what each variable represents.

   (b) This system of equations has an infinite number of solutions. Solve the system of equations using matrices and row reduction. Give the solution in terms of a parameter t, where $t \in \mathbb{R}$.

   (c) In reality, the number of each coin must be a non-negative integer and so we can't use all possible real values of t. How many values of t lead to a valid solution?